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Space-time quantum solves three experimental paradoxes

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ABSTRACT

I show that a Planck-scale deformation of the relativistic dispersion relation, which has been independently considered in the quantum-gravity literature, can explain the surprising results of three classes of experiments: (1) observations of cosmic rays above the expected GZK limit, (2) observations of multi-TeV photons from the BL Lac object Markarian 501, (3) studies of the longitudinal development of the air showers produced by ultra-high-energy hadronic particles. Experiments now in preparation, such as the ones planned for the GLAST space telescope, will provide an independent test of this solution of the three experimental paradoxes.

Theoretical physics has been puzzling over the structure of space-time at distance scales of the order of the Planck length[1] L_p for several decades [1, 2, 3]. Unfortunately, there was no experimental counter-part for these sizeable theoretical effort. All effects predicted by Planck-scale theories are very small, since they are strongly suppressed [2, 3] by the smallness of the Planck length, and this has kept the Planck-length structure of space-time beyond the reach of available experimental sensitivities. Only over the last 15 years some ideas for experimental investigations of this realm have started to emerge (see, *e.g.*, Refs. [4-16]), relying on the remarkable sensitivities of advanced experiments now in preparation.

At a time when it appeared to be rather exciting [2, 3] that some experiments could finally start exploring, in a few years, the structure of space-time at the Planck scale, it was recently argued (see, *e.g.*, Refs. [17, 18, 19]) that we might be already witnessing the first manifestations of Planck-length physics, since quantum-gravity models can provide solutions for some experimental paradoxes that presently confront the astrophysics community: observed violations of the cosmic-ray GZK limit [20] and observed violations of the analogous 10-TeV limit [18, 21] that applies to photons from Markarian 501 (a BL Lac object at a redshift of 0.034, *i.e.* ~ 157 Mpc).

I shall revisit these analyses of observations of cosmic rays and Markarian-501 photons and I shall then consider another independent experimental paradox which emerged from a very recent analysis [22] of data on the longitudinal development of the air showers produced by ultra-high-energy hadrons. Remarkably, I find that all three paradoxes are solved by the same phenomenological model of Planck-length physics, characterized by a deformed dispersion relation (without free parameters!). The preliminary evidence emerging from these three paradoxes could amount to establishing the first Planck-scale property of space-time. Because of the profound significance of such a discovery it is at present necessary to proceed very cautiously, and, accordingly, I shall also emphasize the aspects of these three paradoxes that still require further investigation. The situation will be fully clarified, as I discuss in the final part of this note, by forthcoming observations by the GLAST space telescope [23] which can provide an independent and robust test of the scenario that is being encouraged by the three experiments here analyzed.

Let me start briefly reviewing the three experimental paradoxes. They all involve the kinematic rules for particle production in a continuum classical space-time, but the relevant particle-production processes are different and the energy scales involved are also different:

Cosmic-ray paradox. Cosmic rays can interact with the Cosmic Microwave Background Radiation (CMBR), producing pions. Taking into account the typical energy of CMBR photons, and assuming the validity of the kinematic rules for the production of particles in our present, classical and continuous, description of space-time (conventional relativistic kinematics), one finds that these interactions should lead to an upper limit $E < 5{\cdot}10^{19}$eV, the GZK limit [20], on the energy of observed cosmic rays. Essentially, cosmic rays emitted with energies in excess of the GZK limit should loose energy on the way to Earth by producing pions, and, as a result, should still satisfy the GZK limit when detected in our observatories. Instead, several cosmic-rays above the GZK limit (with energies as high as $3{\cdot}10^{20}$eV) have been observed [24].

Markarian-501 paradox. Just in the same way in which one obtains the GZK limit

[1] $L_p \equiv \sqrt{\hbar G/c^3} \sim 1.6{\cdot}10^{-33} cm$, where $\hbar$ is the reduced Planck constant, G is the gravitational constant and c is the speed-of-light constant

for cosmic rays, one also obtains a limit on the maximum energy of photons that can reach us from distant sources. The relevant process for establishing this limit is pair-production absorption of high-energy photons due to interactions with the Far Infrared Background Radiation (FIBR). For the high-energy photons emitted by Markarian 501, FIBR absorption should [18, 21] become efficient around 10TeV. Markarian-501 photons with higher energies should collide with FIBR photons, disappearing into an electron-positron pair, and should therefore not be able to reach our observatories. Instead, Markarian-501 photons with energies as high as 24TeV have been observed [25].
Pion-stability paradox. The kinematic rules for the production of particles also govern the structure of the air showers produced by high-energy particles. In particular, they allow to predict some features of the longitudinal development of the showers, such as the probability distribution of the maximum depth of the showers. Experimental data on the longitudinal development of the air showers produced by ultra-high-energy hadronic primaries appear to be in disagreement [22] with these predictions. The analysis reported in Ref. [22] suggests that the observed longitudinal development of the air showers could be explained by assuming that ultra-high-energy neutral pions are much more stable than low-energy ones, as if, at ultra-high energies, the available phase space for decay in two photons was becoming smaller (perhaps, at some energy, even vanishingly small [22]) than the one predicted by conventional relativistic kinematics.

As announced, I intend to show that these paradoxes can be solved by adopting a deformation of the standard relativistic dispersion relation $E^2 = p^2 + m^2$. In the quantum-gravity literature there has been discussion of various mechanisms for the emergence of deformed dispersion relations. The most radically new of these scenarios is the one [26] in which a deformed dispersion relation is assumed to emerge as a reflection of the deformed symmetries of a quantum version[2] of (quasi-)flat space-time. Alternatively, deformed dispersion relations can also emerge as a property[3] of the space-time foam background [29], as illustrated by the phenomenological model considered in Ref. [7] and by the analysis of loop-quantum-gravity "weave states" [30] reported in Ref. [11]. In attempts to unify space-time physics with quantum mechanics one can also encounter deformed dispersion relations as a result of the presence of more ordinary (no foam) backgrounds; for example, string theory in certain magnetic-field-like backgrounds admits an effective-theory description in terms of a field theory in noncommutative geometry with associated emergence of deformed dispersion relations (see, *e.g.*, Ref. [31].)

From the analysis reported here below, the careful reader will easily realize that the key ingredient for the solution of the mentioned three paradoxes is a deformation of the dispersion relation. It appears plausible that more than one of the quantum-gravity schemes that motivate the analysis of such deformations would provide solutions of the paradoxes. However, I shall here focus on the space-time-foam phenomenological scheme advocated in Ref. [7], based on the dispersion relation[4]

$$E^2 = f(E, p; m; L_p) \simeq p^2 + m^2 - L_p E p^2 \ . \qquad (1)$$

[2]Of course, quantum versions of Minkowski space-time usually do not enjoy classical symmetries. In fact one of the schemes considered in Ref. [26] turned out to be connected with the κ-Minkowski noncommutative space-time [27, 28].

[3]Space-time foam could play the role of a dispersion-inducing environment [7, 8, 11].

[4]The precise all-order function f is not discussed in Ref. [7], but present and forthcoming experiments are anyway only sensitive to the leading-order correction to the presently-adopted dispersion relation $E^2 = p^2 + m^2$.

In fact, my analysis will be facilitated by the simplicity of this dispersion relation (for example, in other schemes [11, 31] one should worry about a polarization dependence in the analysis of processes involving photons). Moreover, the fact that (1), unlike other proposed deformations of the dispersion relation, has no free parameters (but see closing remarks on alternatives with different sign and magnitude of the deformation term) renders particularly significant the fact that three independent experimental paradoxes find a common solution in this scheme.

Let me start with the cosmic-ray paradox. As first observed by Kifune [17], the deformed dispersion relation (1) would affect the kinematics of particle-production processes, such as photopion production ($p+\gamma \rightarrow p+\pi$, with the usual notation p, γ, π to denote protons, photons and pions respectively) which, as mentioned, is relevant for the cosmic-ray paradox. Combining (1) with equations for the conservation of energy and momentum one finds that a collision between a proton of energy E and a CMBR photon of (much smaller) energy ϵ can produce a pion (and a proton) only if

$$E > \frac{2m_p m_\pi + m_\pi^2}{4\epsilon} + L_p \frac{(2m_p + m_\pi)^3 \, m_\pi^3}{256\,\epsilon^4} \left(1 - \frac{m_p^2 + m_\pi^2}{(m_p + m_\pi)^2}\right) \; , \tag{2}$$

where m_p (m_π) is the proton (pion) mass. The $L_p \rightarrow 0$ limit of this condition of course describes the conventional photopion-production threshold. In spite of the smallness of L_p the correction term turns out to be significant for the cosmic-ray paradox (the magnitude of the correction term is suppressed by L_p but is boosted by the large ratios m_p/ϵ, m_π/ϵ). In fact, one finds [17, 19] that, according to (2), even at $E \sim 3{\cdot}10^{20}$eV photopion production on CMBR photons is still not possible, providing an explanation for the fact that cosmic rays of such high energies are being observed.

As shown in Refs. [18, 19], the Markarian-501 paradox can be explained in a completely analogous manner. Combining (1) with the relevant equations for the conservation of energy and momentum one finds that the process $\gamma + \gamma \rightarrow e^- + e^+$ is only possible if

$$\mathcal{E} > \frac{m_e^2}{\epsilon} + L_p \frac{m_e^6}{8\epsilon^4} \; , \tag{3}$$

where m_e is the electron mass, $\mathcal{E}$ is the energy of the (hard) photon emitted by Markarian 501, and ϵ denotes again the energy of the (soft) background photon (here assumed to be a FIBR photon). The $L_p \rightarrow 0$ limit of (3) of course describes the conventional pair-production threshold. The L_p-dependent term again represents a significant correction ($m_e/\epsilon \gg 1$). Substituting for ϵ some typical energies of FIBR photons (~ 0.005 eV) one finds that the correction term is sufficient to forbid electron-positron pair production even well above $E \sim 20$TeV, consistently with the observations reported in Ref. [25].

The cosmic-ray paradox and the Markarian-501 paradox involve different energy scales and different collision processes, but admit the same type of description (they require an increase in the theory estimate of the threshold energy) and, as shown, admit a common solution based on (1). The pion-stability paradox emerging from the analysis reported in Ref. [22] is of a different type, since it involves a particle-decay process rather than a collision process. While for the cosmic-ray and the Markarian-501 paradoxes solutions based on the deformed dispersion relation (1) had already been discussed in the literature [17, 18, 19], previous studies of schemes leading to (1) had

not noticed the associated emergence of neutral-pion increased stability, and actually had not noticed any implications for particle-decay processes. This is the main technical/theory result reported in the present note, and, remarkably, its phenomenology implications for pion decay into photons are in agreement with the indication that has emerged from the analysis reported in Ref. [22]. Let me focus the analysis of the implications of (1) for particle decay directly on the example relevant for the pion-stability paradox: the process $\pi \rightarrow \gamma + \gamma$ (for other particle-decay processes one can of course proceed in strict analogy). My observation is based on the kinematical condition that establishes a relation between the energy E_π of the incoming pion, the opening angle θ between the outgoing photons and the energy E_γ of one of the photons (the energy E'_γ of the second photon is of course not independent; it is given by the difference between the energy of the pion and the energy of the first photon). This relation is found, as usual, by combining the dispersion relation, here assumed to be described by (1), with the equations for the conservation of energy and momentum. One finds

$$\begin{aligned} m_\pi^2 &= [2E_\gamma E'_\gamma + L_p E_\pi E_\gamma E'_\gamma][1-\cos(\theta)] + 2L_p E_\pi E_\gamma E'_\gamma \\ &= [2E_\gamma(E_\pi - E_\gamma) + L_p E_\pi E_\gamma (E_\pi - E_\gamma)][1-\cos(\theta)] + 2L_p E_\pi E_\gamma (E_\pi - E_\gamma) \ . \end{aligned}$$

In the $L_p \rightarrow 0$ limit (the limit that corresponds to our present classical picture of space-time) this kinematical condition of course reproduces the corresponding result for conventional relativistic kinematics. The L_p-dependent correction term starts to be significant at pion energies of order $(m_\pi^2/L_p)^{1/3}$. When $E_\pi^3 > 2m_\pi^2/L_p$ one finds that some of the values of E_γ which correspond to viable decay processes according to conventional relativistic kinematics are no longer available to the decay process (in particular, since $1-\cos(\theta)$ is always positive, one must exclude all values of E_γ such that $m_\pi^2 - 2L_p E_\pi E_\gamma (E_\pi - E_\gamma) < 0$). As one easily sees from (4), this reduction of the available phase space starts rather quietly (only a very small reduction) at $E_\pi \simeq (2m_\pi^2/L_p)^{1/3} \sim 10^{15}$eV, but gets stronger and stronger as the pion energy increases. This picture of pion decay, with the associated depletion of the number of photons produced by ultra-high-energy neutral pions, would explain the puzzling experimental data discussed in Ref. [22].

Having added the pion-stability paradox to the cosmic-ray and Markaria-501 paradoxes, we now have three paradoxes that are solved by adopting the Planck-scale-deformed dispersion relation (1). In the literature one does not find any other indication of departures from a classical space-time picture and it is easy to check [7, 17] that in the (huge number of) experiments that are consistent with the classical dispersion relation $E^2 = p^2 + m^2$ the correction term introduced in (1) is completely negligible (in order to compensate for the smallness of L_p the physical context must involve unusually large hierarchies between some relevant energy/length scales, such as the ratios m_p/ϵ, m_e/ϵ and E_π/m_π encountered in the analysis of the paradoxes). Therefore, we seem to be confronted with the exciting perspective of having to replace the classical space-time picture with a new (quantum) picture involving the Planck length. I should stress however that, while ordinarily the combined indications of three independent experiments are considered to be sufficient for drawing definitive conclusions, the three experiments on which my analysis is based are still subject to some residual elements of doubt, and some prudence may be appropriate.

The cosmic-ray paradox is well established, in the sense that there can be no residual doubt concerning the fact that cosmic rays with energies beyond the GZK limit are

being detected. There are however some alternative (not less speculative [17]) possible explanations of the cosmic-ray paradox, which exploit the fact that we are unable to identify the astrophysical sources of these cosmic rays and that the identification of ultra-high-energy cosmic ray as protons is still subject to (however small [22]) margins of uncertainty.

With respect to the Markarian-501 paradox the residual elements of doubt are complementary to the case of cosmic rays. In fact, we have a clear identification of the observed particles as photons and equally clear is the identification of Markarian 501 as the source. However, while measurements of the CMBR have become more and more accurate over the years, measurements of the FIBR have reached a satisfactory level of accuracy only very recently (see, *e.g.*, Ref. [32]) and the robustness and interpretation of these recent experimental results may still be subject to further scrutiny. This is of course significant for establishing the Markarian-501 paradox, since the likelyhood that a Markarian-501 photon above threshold would reach our detectors also depends on the (density of the) FIBR.

Concerning the robustness of the pion-stability paradox, besides the need for more accurate data (some of the graphs that support the analysis reported in Ref. [22] show data points with significant error bars), a key reason of residual concern resides, in this author's opinion, in the role that quantum chromodynamics (QCD) playes in the structure of the air showers produced by hadronic particles. The analysis reported in Ref. [22] appears to provide rather convincing evidence of the fact that within the presently-favoured phenomenological model of the relevant QCD processes (a model which has been found to be reliable in other contexts) the assumption of increased pion stability at ultra-high energies leads to improved agreement with the data on the longitudinal development of the air showers. However, certain quantitative estimates based on QCD are still rather challenging (in spite of the fact that QCD has been well understood conceptually and at the level of the formalism for several years) and it appears to be reasonable to wonder whether one should also explore the possibility of modifying the presently-favoured phenomenological model of QCD processes, without introducing an increase in pion stability.

In summary, the status of the three paradoxes is rather robust, but each of them (to different degrees) is still not completely immune from potential weaknesses. Perhaps a greater level of confidence should be attributed to the analysis here being reported by considering the consistency of the combined indications of the three experiments. In particular, from a strictly phenomenological viewpoint one could also contemplate deformed dispersion relations with the same structure of (1) but with the opposite sign choice for the correction term and/or with a deformation scale which is significantly different from the Planck scale. But for all three paradoxes the solutions require the same sign choice[5], so in order to assume that the evidence emerging from these paradoxes is the result of the preliminary nature of the experimental data one should assume that the independent inaccuracies of these data have somehow conspired to point all in the same direction. Similarly, the evidence emerging from the three paradoxes also has a significant level of internal consistency for what concerns the deformation length scale.

[5]With the opposite sign choice the two threshold conditions here considered would go in the opposite direction, predicting that the process is allowed at even lower energies than in the conventional theory (in clear contrast with the experimental information). As one can easily see by changing the sign in front of L_p in (4), the opposite sign choice would also not predict the increase of pion stability here discussed.

It is easy to see that the requirement of explaining the three paradoxes imposes that this length scale cannot be much smaller than L_p (the solution of the Markarian-501 paradox is lost already by decreasing the deformation length scale by a factor of 100 or so, while the solutions of the cosmic-ray paradox and of the pion-stability paradox have a few more orders of magnitude margin). On the other hand the deformation length scale certainly cannot be much larger than L_p because otherwise a disagreement would emerge with data at lower energies [7]. Therefore the requirement of explaining the three paradoxes, besides imposing a consistent sign choice, also imposes that the deformation length scale be within a few orders of magnitude of the Planck length, just as one would expect in light of the quantum-space-time arguments that support (1).

While some prudence is certainly appropriate, we are clearly confronted with growing experimental evidence in favour of the exciting perspective of having to modify our present classical description of the short-distance structure of space-time. The issue will be completely settled within a few years by experiments such as the ones planned for the GLAST space telescope. As discussed in detail in Refs. [7, 10, 23] these experiments are sensitive to the implications of (1) for the structure of bursts of gamma rays that we detect from distant astrophysical sources, an effect that (since it does not involve particle-production processes) is completely independent from the effects here analyzed in association with (1). The expected sensitivity levels of GLAST (which even extend several orders of magnitude beyond the Planck-length choice of the deformation length scale) are such that (1) will be either fully confirmed or completely rejected [23].

If the GLAST verdict does confirm the growing evidence that is emerging from the experimental paradoxes here considered, theoretical physics will find itself in a situation that is amusingly analogous to the one that was created, a century ago, by the Michelson-Morley experiments (which can be described as providing evidence in favour of the dispersion relation $E^2 = p^2$ for photons, and forced a revolution in the description of space-time, abandoning the Galileo-Newton picture). This will include the need to establish which of the space-time pictures that support deformed dispersion relations is actually realized in Nature. At present the fact that (1), without any free parameter, explains all observations appears to be significant. But even if (1) is indeed realized in Nature we will still have to consider two main alternative scenarios that can lead to (1). The scenario on which I focused here, which, as mentioned, is based [7, 10, 17] on some conjectured properties of space-time foam, is strongly characterized by the fact that space-time foam could have a preferred frame [7, 8, 11, 15], just like the classical aspects of the geometry of the space-time of our Universe are such that it is possible to identify a preferred frame[6] (a convenient frame for most applications is the one in which the CMBR has the simplest properties [17]). The second scenario that supports (1) is based [26] on the opposite assumption: the quantum features of space-time would not have a preferred frame, in which case the role of L_p in (1) should be enforced as an observer-independent condition. As mentioned, this second scenario leads [26] to a fundamental picture of space-time that is noncommutative [27, 28], and accordingly Lorentz transformations between different frames would be governed by a quantum-algebra version of the Lorentz algebra that had emerged [33, 28] as part of

[6] At the fundamental level the theory does not have a preferred frame, but of course the field distributions that correspond to a given solution of the equations of dynamics can be used to identify a preferred frame (a frame in which these field distributions acquire a certain characteristic form.)

the mathematical-physics programme of systematic studies of quantum deformations of classical groups and algebras.

It is rather compelling that the simple first scenario, here considered, manages to explain the three paradoxes, in spite of its highly constrained structure (no free parameters). Still, it will of course be interesting to compare the paradoxes with other schemes leading to deformed dispersion relations, and particularly the mentioned scenario [26] that supports (1) as an observer-independent property of (noncommutative) space-time. This more delicate (both technically, because of the complexity of noncommutative geometry, and phenomenologically) analysis is postponed to future studies [34].

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